Calculation of Power Output and Quantification of Training Stress in Distance Runners: The Development of the GOVSS Algorithm

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Introduction:

Exercise professionals have long sought the ability to quantify stress or training load on athletes. This desire is borne of the observation that athletes exhibit specific responses to particular stimuli. For example, asking a runner to train 20 kilometers per week may improve fitness slightly, 40 kilometers somewhat more, but 60 kilometers might result in injury. As might be intuitively apparent, many sporting injuries (particularly in endurance sports) are the result of training error (O’Toole 1992). With this in mind, an objective system quantifying training stress would be helpful in planning and monitoring not only the training and development, but the health of the athlete.

There exists a dose-response relationship between training stimulus and adaptation of the subject (Bannister et al 1975, Busso 2003). Training load can be expressed simply as:

\[ \text{Training load} = \text{Intensity} \times \text{Duration} \] (Eq. 1)

It is clear that different stimuli will have different physiologic sequelae. It is less clear how to compare/quantify differing stimuli and their ability to affect the same response. An oversimplified example might be: Would a twenty minute run at 10 kph cause the same physiologic strain as 10 minutes at 20 kph? Sports professionals may attempt to make such comparisons since the relationship between oxygen uptake/heart rate and velocity/work rate is essentially linear up to VO2max (Astrand et al 2004). However, other relevant relationships are not linear; that between velocity and metabolic stress (i.e. lactate production) is exponential in nature (Astrand et al 2004, Coyle 1999, Farrell et al 1978). Also, submaximal heart rate (HR) and oxygen uptake are dependant upon work rate, not the other way around.

We now understand that irrespective of heart rate or absolute oxygen uptake, the work rate at lactate threshold (defined as a rise of serum lactate of 1 mmol/L over exercise baseline) is the primary determinant of endurance exercise performance (Coyle 1988, 1999). The nonlinear relationship between lactate and training stress was understood by E.W. Banister as early as 1975. At that time, there was no easy/direct means of quantifying work rate objectively in the field. Thus, he sought to relate an easily measured parameter (heart rate) to lactate production through the use of a population study. He devised a system of measurement known as TRIMPS, or TRaining IMPulse Score.

\[ \text{TRIMPS} = \text{Duration} \times \text{Average HR during exercise} \times \text{A HR-dependant, intensity based weighting factor} \] (Eq. 2)

The benefit of Banister’s system is that it takes into consideration the observation that higher workloads are more metabolically taxing (exponentially so) than lower workloads of equivalent duration (Bannister 1996). However, it is still dependant upon the measurement of heart rate, which is variable based on factors such as hydration, rest, illness, or cardiac drift. Furthermore, though HR is dependant upon workload, it may take minutes to stabilize when that workload changes. It was still necessary to find a way to quantify the work directly.

In 2003, Andrew Coggan refined Banister’s concept by developing a system that also incorporated lactate response to workload. This system related the change in lactate concentration with the change in an objective measure of exercise intensity: power output. The relative benefits of this system are immediately apparent: power meters are extremely accurate, are present on cycle ergometers in most gym settings, and may be installed on standard bicycles. The power meter provides a direct and immediate measure of work rate; there is no need to wait several minutes for heart rate or oxygen uptake to stabilize.

Coggan devised a mathematical algorithm similar to that of Bannister, called the Training Stress Score (TSS).
TSS = Exercise duration \cdot Average \text{ power} \cdot \text{Power-dependant, intensity weighting factor} \ (\text{Eq. 3})

The power dependant intensity weighting factor was derived directly from a plot of blood lactate concentration as a percentage of concentration at threshold against % of threshold power. His work indicated a near 4\textsuperscript{th} power relationship between the two.

\begin{align*}
\text{Blood lactate (\% of lactate at Lactate Threshold) =} \\
\text{Power (\% power @ LT)}^{3.96}, \ R^2=0.806, \ n=76. \ (\text{Eq. 4})
\end{align*}

It should be noted that Coggan (2003, 2006) has advocated using a 30 second rolling average to smooth the power data and facilitate analysis, which is sensible given the many physiologic processes have 30 second half-lives (e.g. HR, plasma epinephrine concentration, ventilation, etc).

The TSS is predicated upon the concept of Normalized Power (NP). Coggan developed this concept to address a point intuitively obvious to anyone who has trained using a power meter: the average power (AP) for an exercise task is not necessarily indicative of athlete strain unless the task in question is a roughly isopower effort. For example, a one hour time-trial (TT) in flat terrain may be undertaken at a maximal / exhaustive effort which results in an AP of 300 watts, with little variation in power output (Figure 1a). Yet, a 1 hour criterium-type race that involves surges in pace and periods of coasting or a hilly TT may also be exhaustive, yet result in an AP of only 225 watts (Figure 1b). By weighting power output with a fourth power function, it becomes possible to make more direct comparisons between exercise tasks; utilizing Coggan’s 4\textsuperscript{th} power weighted average to “normalize” power output, the TT effort remains about 300 normalized watts, and the variable effort of 225 watts becomes 301 normalized watts (Figure 1a and b). As might be expected, NP for a variable power, maximal effort one hour TT or criterium has been shown to be more highly correlated to the AP obtained in a one hour maximal isotrace TT than AP for the variable power effort is (p=.001 and p=.04, n=5) (Skiba, 2006. In review).

The elegance of Coggan’s system is that while it successfully relates lactate concentration to power output, it is not dependant upon invasive tests. In 1988, Coyle et al. illustrated that the highest power output or pace an athlete can maintain over the course of an hour long exercise task is highly correlated with LT (Coyle et al. 1988). Thus, to determine threshold intensity, the athlete need only perform such a test and use the resulting average power in the calculations. The concept carries over to the running literature as well, where 8k-10k to 1 hour runs at the maximal pace sustainable for the duration of the run have been shown to be strongly correlated to both LT and maximal lactate steady state (MLSS); that is, the highest exercise intensity that does not result in a continual increase in serum lactate (Jones and Doust 1998, Daniels 2002).

Analysis has remained more difficult with regard to running, because it is impossible to wire a power-meter to a human being. Thus, runners have been limited to the use of Banister’s HR based methods for the determination of training stress, load, and response, with all of the associated disadvantages. However, recent technological advances (i.e. wrist-top altimeters/GPS receivers, shoe-worn accelerometers) have made it possible to easily and objectively measure and record the elements which can be used to calculate power output. This work will demonstrate the means to do exactly this and to apply a system of training stress quantification to the data.
Methods:

Part I: Relevant mathematics

While a runner’s power output is not directly measurable without a treadmill ergometer, several groups have described a power-balanced supply-demand approach to running energetics. Let $C$ = the energy cost of moving forward, $C_{aero}$ = the energy cost of overcoming aerodynamic drag, and $C_{kin}$ the energy cost of changes in velocity.

$$E_{aerobic} \cdot t^{-1} + E_{anaerobic} \cdot t^{-1} = C \cdot V + C_{aero} \cdot V + C_{kin} \cdot V \quad (\text{Eq. 5})$$

This equation has been used successfully in the prediction of performances for middle and long distance running, where $V$ = distance / time (DiPrampero et al 1986). In this model, $C_{aero} = k \cdot n^{-1} \cdot d^2 \cdot t^2$, and $k$ is the constant of air friction (in kg$^{-1} \cdot$ m$^{-1}$) with $n = 0.5$. $C_{kin} = 0.5 \cdot n^{-1} \cdot d \cdot t^2$, with $n = 0.25$. It has also been minimally modified to accurately predict performance in events ranging from 800M to 5k (DiPrampero et al 1986, 1993. Arsac et al 2001).

It has been determined that the energy requirement ($C$) to cover any given distance is essentially independent of velocity, is consistent within individual athletes, and is equivalent to between 3.6-4.2 J·Kg$^{-1} \cdot$ M$^{-1}$. (Margaria et al 1963, Pugh 1970, Cavagna and Kaneko 1976, Fellingham et al 1978). Importantly, $C$ also varies with the slope of the running surface (i) according to a 5$^{th}$ order polynomial regression (Minetti 2002):

$$C_i = 155.4i^5 - 30.4i^4 - 43.3i^3 + 46.3i^2 + 19.5i + 3.6; \text{n}=10, \text{R}^2 = .999 \quad (\text{Eq. 6})$$

It should be noted that $C$ describes the total energetic requirement, and does not address the efficiency of conversion from metabolic to external power available for locomotion. However, this is easily rectified by multiplying by an efficiency factor ($n_v$), which numerous groups have demonstrated increases with velocity (Lloyd and Zacks 1971, Cavagna and Kaneko 1976, Harris et al 2003). This is due to the larger contribution of passive elastic return of energy from soft tissues with increased velocity, increasing from perhaps 0.25 to 0.35 at the slowest running speeds to approximately 0.5-0.7 at 8.33 m/s (30 km/h) in a reasonably linear fashion (Cavagna and Kaneko 1976, Arsac 2001). This improved efficiency must be adjusted for to derive the amount of contractile energy the athlete must expend to maintain a given velocity. Equation 5 therefore becomes:

$$\text{Power} \ (W/kg) = (C_i \cdot n_v) - (C_i \cdot n_v \cdot (0.5 \cdot (V \cdot 8.33^{-1})))) + C_{aero} \cdot V + C_{kin} \cdot V \quad (\text{Eq. 7})$$

Where all values of $C$ are calculated as rolling averages over 120 second intervals to account for the fact that the original model was validated to the 800M distance (time of slightly less than 2 minutes), and $n_v$ is the efficiency calculated at velocity $V$. 
Part II: Data analysis

A. Lactate/Velocity Data

In order to discern the precise correlation velocity/power and lactate concentration, mean lactate vs. velocity data for runners were obtained from data previously reported by Held and Marti (Held and Marti 1995). The experimental subjects were members of the Swiss national teams in several sports. The athletes were exercised via a standard protocol on a treadmill. Beginning at approximately 3.5 m/sec, treadmill velocity was maintained for three minutes, after which there was a 30 second pause to facilitate collection of a capillary blood sample from the ear lobe. Treadmill speed was then increased by 0.5 m/sec, and the process was repeated until volitional fatigue.

To ensure the results were applicable to well-trained endurance athletes, the top 10% of performances were analyzed. These amounted to 32 trained male runners and 15 trained female runners. Standard deviation in maximal speed was 0.2 m/sec for men and 0.3 m/sec for women (Held and Marti 1995). Velocity/power at LT was deduced by measurement at the first rise of lactate of 1 mmol above exercise baseline. In order to facilitate comparisons between data sets, the velocities and equivalent power outputs were plotted as percentages of their respective values at LT, and regressions were generated using Microsoft Excel. In order to make a comparison with less-aerobically fit athletes, this analysis was repeated with the 10% worst performers reported by the group.

The data from each individual analysis was then plotted together, and a regression was calculated that would apply to the greatest number of trained athletes.
Results:

The relationship between velocity/power and lactate concentration in the top 10% of runners was best described by an exponential function. This is not surprising given the number of investigators who have reported this in the past (Farrell et al 1979, Hermansen and Stensvold 1972, Saltin and Karlsson 1971). However, a power function resulted in a similar fit and simplified the following mathematics.

\[
\begin{align*}
\text{Men (Best 10%):} & \quad Y_{\text{Lactate}} = X^{4.2925} \quad \text{Power (\% of Power at LT)}, \quad N=32, \quad R^2=.9062 \\
\text{Women (Best 10%):} & \quad Y_{\text{Lactate}} = X^{4.5971} \quad \text{Power (\% of Power at LT)}, \quad N=15, \quad R^2=.9454 \\
\end{align*}
\]

(Eqs. 11a and 11b; Figures 2a and 2b)

The relationship between velocity/power and lactate concentration in the worst 10% of runners was also best described by an exponential function, with a similar fit from a power function.

\[
\begin{align*}
\text{Men (Worst 10%):} & \quad Y_{\text{Lactate}} = X^{2.5441} \quad \text{Power (\% of Power at LT)}, \quad N=32, \quad R^2=.9909 \\
\text{Women (Worst 10%):} & \quad Y_{\text{Lactate}} = X^{2.4913} \quad \text{Power (\% of Power at LT)}, \quad N=15, \quad R^2=.9846 \\
\end{align*}
\]

(Eqs. 12a and 12b; Figures 3a and 3b)

A regression was then calculated across all data sets.

\[
\begin{align*}
\text{All Athletes:} & \quad Y_{\text{Lactate}} = X^{3.5249} \quad \text{Power (\% of Power at LT)}, \quad N=94, \quad R^2 = 0.865 \\
\end{align*}
\]

(Eqn. 14; Figure 4)
Discussion:

As expected, all curves fit to the lactate/velocity plots exhibited an exponential pattern as has been observed in the literature for many years. To make the following mathematics easier, relationships were derived using a best fit power function as was done by Coggan, which also provided a good model as seen in the results section. Interestingly, the relationship of lactate concentration to velocity changed from near a 4th power correlation in the athletes of highest fitness to a squared function in the less aerobically fit athletes. This relationship appears to be preserved with relationship to other data sets, and most likely reflects improvements in lactate clearance with increasing athlete fitness (Coggan AR, private communication).

Coggan reported similar data in 2003; a near fourth-power correlation. However, his data set was not reported in terms of the relative fitness of the athletes. Rather, he chose to consider the athletes as a whole to find a function applicable to the most number of people (Coggan AR, private communication). This approach seems sensible, and was repeated here for precisely this reason.

Utilizing a scheme similar to Coggan’s, it is now possible to calculate lactate-normalized power output values and corresponding training stresses given the appropriate recorded values from an exercise bout. The algorithm is as follows:

1. Find the athlete’s velocity at LT by a 10 km to one hour maximal run.
2. Convert this velocity to a power value using Equation 7.
3. Analyze the data from a particular workout from an athlete’s log, computing 120 second rolling averages from velocity and slope data.
4. Raise the values in step 3 to the 4th power.
5. Average values from step 4.
6. Take the 4th root of step 5. This is the Lactate-Normalized Power.
7. Divide Lactate Normalized Power by Threshold Power from step 2 to get the Intensity Weighting Fraction.
8. Multiply the Lactate Normalized Power by the duration of the workout in seconds to obtain the normalized work performed in joules.
9. Multiply value obtained in step 8 by the Intensity Weighting Fraction to get a raw training stress value.
10. Divide the values from step 9 by the amount of work performed during the 10k to 1 hr test (threshold power in watts x number of seconds).
11. Multiply the number from step 10 by 100 to obtain the final training stress in GOVSS™ (Gravity Ordered Velocity Stress Score).

As was intimated earlier, this system is easily realized given that wrist-top GPS receivers are now inexpensive and directly measure and record all necessary variables. Furthermore, their output is readily downloadable to various software packages for analysis. The calculations can be carried out utilizing a spreadsheet program such as Microsoft Excel. (A worksheet automating these calculations is available at http://www.physfarm.com).

The utility of this work is clear. The GOVSS algorithm allows comparison between different training tasks (i.e. hilly vs. flat runs, short/intense interval efforts vs. longer, steadier paced efforts). This permits a more accurate measurement of athlete training stress, and should facilitate analysis of athlete performance with respect to that stress. Furthermore, in the case of injury, it is possible to quantify the amount of stress / rate of stress increase over time that yielded the injury, and therefore avoid it in the future.

There are shortcomings to this means of calculating the energy cost of running. The first is that the absolute energy requirement will be slightly different for each person given their personal characteristics (the athlete’s individual energy cost of running or absolute efficiency, for instance).
However, because these sources of error are apparently consistent, it will affect the accuracy but not the precision of the model. Another important consideration is that the aerodynamic cost of running is not adjusted for temperature or barometric pressure. However, this did not preclude the validity of the model in the work of DiPrampero et al (1986, 1993). Furthermore, it is important to realize that both $C_{aero}$ and the term describing the change in kinetic energy are extremely small at typical endurance running velocities, with $C_{aero}$ describing 8-10% and $C_{kin}$ approximately 1% respectively of the total power requirement (Arsac 2001).

Pilot studies have indicated that the GOVSS algorithm above permits the quantification of training stress on a more replicable basis than TRIMPS (Skiba 2006, in press) (Figure 5). Furthermore, GOVSS has been shown to be an acceptable input function for systems based performance prediction equations such as those developed by Banister, Fitz-Clarke and Morton (1990, 1991) (Skiba 2006, in press). Work is in progress which focuses on a comparison between TRIMPS and GOVSS as input functions into these performance prediction equations.
Figures 1A and 1B: Graphical comparison between average power (AP, dashed line) and Normalized Power (NP, solid line) for isopower and variable power maximal time trial efforts (top and bottom, respectively). Power output is recorded as a 30 second rolling average. Note that while AP changes between efforts, normalized power remains essentially constant. Data from: Skiba, 2006 (in review).
Figures 2a and 2b: Graphical representation of the relationship between velocity and lactate concentration as percentages of respective values at LT for males (top, N=32) and females (bottom, N=15) in the top 10% of athletes. Data from: Held and Marti, 1995.
Figures 3a and 3b: Graphical representation of the relationship between velocity and lactate concentration as percentages of respective values at LT for males (top, N=32) and females (bottom, N=15) in the bottom 10% of athletes. Data from: Held and Marti, 1995.
Figure 4: Graphical representation of the relationship between velocity and lactate concentration as percentages of respective values at LT for all athletes, N=94. Data from: Held and Marti, 1995.
Figure 5: Graphical representation of the relationship between TRIMPS (gray) and GOVSS (black) for the same 6.5k run conducted on different days. Run 3 was conducted at an environmental temperature 10 degrees centigrade warmer than Runs 1 and 2. Data from: Skiba, 2006 (in press).
References:


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