

# Modeling the energetics of 100-m running by using speed curves of world champions

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<sup>1</sup>Faculté des Sciences du Sport, Université Victor Ségalen Bordeaux 2, Domaine Universitaire, 33607 Pessac cedex; and <sup>2</sup>International Amateur Athletic Federation, 98007 Monaco cedex, France

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**Arsac, Laurent M., and Elio Locatelli.** Modeling the energetics of 100-m running by using speed curves of world champions. *J Appl Physiol* 92: 1781–1788, 2002. First published November 30, 2001; 10.1152/jappphysiol.00754.2001.—The present study aims to assess energy demand and supply in 100-m sprint running. A mathematical model was used in which supply has two components, aerobic and anaerobic, and demand has three components, energy required to move forward (C), energy required to overcome air resistance (Caero), and energy required to change kinetic energy (Ckin). Supply and demand were equated by using assumed efficiency of converting metabolic to external work. The mathematical model uses instantaneous velocities registered by the 1997 International Association of Athletics Federations world champions at 100 m in men and women. Supply and demand components obtained in the male champion were (in J/kg) aerobic 30 (5%), anaerobic 607 (95%), C 400 (63%), Caero 83 (13%), Ckin 154 (24%). Comparatively, a model that uses the average velocity of the male and female 100-m champions overestimates Ckin by 37 and 44%, respectively, and underestimates Caero by 14%. We argued that such a model is not appropriate because Ckin and Caero are nonlinear functions of velocity. Neither height nor body mass seems to have any advantage in the energetics of sprint running.

energy cost of running; acceleration; air resistance; anaerobic energy; body size

IT HAS BEEN PROPOSED that the energetics of running might be appropriately described by using a power-balanced model based on a supply-demand approach (11, 21, 22, 25–27). In such a model, the supply side has two components (aerobic and anaerobic) and the demand side has three components [energy required to move forward (C), energy required to overcome air resistance (Caero), and energy required to change kinetic energy (Ckin)]

$$\begin{aligned} E_{\text{aer}} \cdot t^{-1} + E_{\text{ana}} \cdot t^{-1} \\ = C \cdot V + \text{Caero} \cdot V + \text{Ckin} \cdot V \end{aligned} \quad (1)$$

where  $E_{\text{aer}}$  and  $E_{\text{ana}}$  (in J/kg) represent the amount of energy released by aerobic and anaerobic processes, respectively, over the time period  $t$  (in s); C, Caero, and Ckin are expressed in  $\text{J} \cdot \text{kg}^{-1} \cdot \text{m}^{-1}$ , and the running

velocity is  $V$  (in m/s). Most authors (10, 11, 21, 22, 25), but not all (26, 27), include in Caero and Ckin the concept of efficiency ( $\eta$ ) of converting external work to its metabolic energy equivalent. C, however, has a metabolic dimension in the literature (3, 10, 11, 16, 21, 24) so that no assumption should be made about  $\eta$ .

Concerning demand, the relative contribution of each component is very different depending on the running distance. The longer the running distance, the lower the contribution of Ckin and Caero (11). Thus, for short-distance running, such as a 100-m race, both Ckin and Caero make significant contributions to demand.

The most developed application of *Eq. 1* concerns middle- and long-distance running in which the average running velocity can be approximated by  $d/t$ , where  $d$  is the known distance covered (in meters) and  $t$  the measured running time (in s). In the model that uses the average velocity ( $d/t$ ),  $\text{Caero} = k \cdot \eta^{-1} \cdot d^2 \cdot t^{-2}$ , where  $k$  is the air friction constant (in  $\text{kg}^{-1} \cdot \text{m}^{-1}$ ) and  $\eta = 0.5$  (11, 21, 22), and  $\text{Ckin} = 0.5 \cdot \eta^{-1} \cdot d \cdot t^{-2}$ , where  $\eta = 0.25$  (11, 21, 22).

In principle, this analytical model employed for the mathematical analysis of a wide range of running performances (21) can be applied to investigate the results of individual athletes. This has been done successfully by di Prampero and colleagues (10), who demonstrated good relationships between actual and predicted performances in 36 individuals at marathon and half-marathon distances (42 and 21 km) by using average velocity in the model. For such long-distance races, demand is almost entirely described by C (92%) because Caero is low in still air,  $\sim 0.3 \text{ J} \cdot \text{kg}^{-1} \cdot \text{m}^{-1}$  or 8% of the total demand, and Ckin is negligible.

Such reliability of results was also obtained (11) over shorter distances (0.8–5 km) in 16 runners of intermediate level and 27 elite athletes tested by Lacour and colleagues. The model that uses average velocity indicates that running 0.8 km in world record time (101.11 s) would mean that  $\text{Caero} = 0.63 \text{ J} \cdot \text{kg}^{-1} \cdot \text{m}^{-1}$ , or 14% of the total demand, and  $\text{Ckin} = 0.16 \text{ J} \cdot \text{kg}^{-1} \cdot \text{m}^{-1}$ , or 3% of the total demand.

Also by using the average velocity, Peronnet and Thibault (21) obtained a power-balanced *Eq. 1* for a

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100-m sprint covered in 9.95 s. Aerobic and anaerobic supply [42 J/kg (6%) and 650 J/kg (94%) above resting, respectively] were balanced with  $C = 386$  J/kg (56%),  $Caero = 104$  J/kg (15%), and  $Ckin = 202$  J/kg (29%). However, it is worth noting that, in a 100-m race, the initial acceleration phase is so important that average velocity is not appropriate when calculating  $Ckin$ . Considering  $Ckin$  as  $0.5 \cdot \eta^{-1} \cdot V^2 \cdot d^{-1}$  would be strictly correct only if the final velocity attained during the race is equal to the average velocity. This is far from being the case in sprinting. There is also a problem with  $Caero$  for the same reason:  $Ckin$  and  $Caero$  are nonlinear functions of running velocity. As pointed out by Capelli et al. (5), when analyzing the energetics of cycling over short distances, an improved energetic model could be achieved when instantaneous velocities, rather than average velocity, throughout the race is considered.

An alternative mathematical procedure to that of model  $d/t$  can be identified in the literature (25) in which supply and demand are equated by using instantaneous velocities  $V_t$ . When using the model  $V_t$ , Eq. 1 becomes

$$Paer_t + Pana_t = C \cdot V_t + Caero_t \cdot V_t + \Delta Ekin \cdot \Delta t^{-1}, \quad (2)$$

where  $Paer_t$  and  $Pana_t$  (in W/kg) represent instantaneous aerobic and anaerobic power, respectively;  $Caero_t$  (in  $J \cdot kg^{-1} \cdot m^{-1}$ ) is the instantaneous  $Caero$ ; and  $\Delta Ekin$  (in J/kg) is obtained from changes in kinetic energy:  $0.5V_{(t+1)}^2 - 0.5V_t^2$ . For solving Eq. 2,  $V$  at any instant  $t+1$  is calculated by numerical integration and as a function of the precedent  $V$  at instant  $t$  (25).

The model  $V_t$  was used with assumed supply and calculated demand so that the speed curve and running performance were predicted (25). It was also used inversely with measured distance-time data of the 100-m sprint and predicted kinetics of anaerobic metabolism (27).

For the first time in Athens, on the occasion of the 1997 International Association of Athletics Federations (IAAF) World Championships in Track and Field, laser apparatus were used to obtain the speed curves of world-class sprint runners (4). For example, the speed curves of both the male 100-m world champion (MWC) and female 100-m world champion (FWC) were registered.

The aim of the present study was to assess energy supply and demand in 100-m world champions, considering  $Caero$  and  $Ckin$ . Because  $Caero$  and  $Ckin$  are nonlinear functions of the running velocity, our hypothesis is that a model, which uses instantaneous velocities rather than average velocity, can provide more valid estimated results. For a complete list of terms and their default values, refer to Table 1.

## METHODS

Van Ingen Schenau et al. (25) first proposed the theoretical model we used.

*Supply in the model.* It was assumed that the kinetics of the aerobic pathway above resting follow the characteristics of a first order system from the initiation of a vigorous exercise (25) according to  $Paer_t = MAP(1 - e^{-t/\tau_1})$ , where  $Paer$  (in W/kg) represents the aerobic power above resting,

Table 1. Glossary

Symbol	Definition	Units	Default Value
Af	Frontal area	m <sup>2</sup>	
C	Energy cost of moving forward (above resting)	J · kg <sup>-1</sup> · m <sup>-1</sup> or J/kg	4
Caero	Energy cost due to aerodynamic resistance	J · kg <sup>-1</sup> · m <sup>-1</sup> or J/kg	
Cd	Drag coefficient		0.9
Ckin	Energy cost due to changes in the body's kinetic energy	J · kg <sup>-1</sup> · m <sup>-1</sup> or J/kg	
d	Running distance	m	
η	Efficiency to convert metabolic into external work		
Eaer	Aerobic energy	J/kg	
Eana	Anaerobic energy	J/kg	
FWC	Female world champion (Marion Jones; weight = 64 kg, height = 1.78 m)		
k	Air friction constant	kg <sup>-1</sup> m <sup>-1</sup>	
MAP	Maximal aerobic power above resting	W/kg	18.4
model $d/t$	Model that uses average running velocity		
model $V_t$	Model that uses instantaneous velocities		
MWC	Male world champion (Maurice Greene; weight = 75 kg, height = 1.75 m)		
Paer	Aerobic power above resting	W/kg	
Pana	Anaerobic power	W/kg	
Pb	Barometric pressure	Torr (kPa)	
Pmax	Maximal anaerobic power	W/kg	
ρ	Air density	kg/m <sup>3</sup>	
T°	Air temperature	°C	
τ <sub>1</sub>	Time constant for reaching MAP at the onset of supramaximal exercises	s	26
τ <sub>2</sub>	Time constant for anaerobic energy release	s	
V	Running velocity	m/s	
V <sub>max</sub>	Maximal running velocity	m/s	
w	Wind velocity	m/s	
WR	2001 World Record		
	Men	s	9.79
	Women	s	10.49

MAP is the subject's maximal aerobic power above resting (in W/kg), and  $\tau_1$  (in s) is the time constant for reaching MAP at the onset of supramaximal efforts. A MAP of 18.4 W/kg was obtained by van Ingen Schenau et al. (25) in sprinters of intermediate level who had a  $\tau_1$  of 26 s, which was in close agreement with Ward-Smith and colleagues (26, 27) and Péronnet and colleagues (21, 22).

In existing models of the energetics of sprinting, the anaerobic power under conditions of all-out exercise is represented by  $P_{ana_t} = P_{max} \cdot e^{-t/\tau_2}$ , where  $P_{max}$  represents the maximal anaerobic power (in W/kg) and  $\tau_2$  (in s) is the parameter governing the rate of anaerobic energy release.

Therefore, in the present analysis, supply, which includes aerobic and anaerobic components, is represented by

$$\text{Supply} = 18.4(1 - e^{-t/26}) + P_{max} \cdot e^{-t/\tau_2} \quad (3)$$

where MAP = 18.4 W/kg and  $\tau_1 = 26$  s have fixed numerical values on the basis of a previous work (25), whereas  $P_{max}$  and  $\tau_2$  are unassigned and are determined by using speed curves measured in world champions.

*Demand in the model.* Demand includes parameters with numerical values fixed on the basis of prior knowledge: C, k, and  $\eta$ .

C, in Eqs. 1 and 2, has been measured in a wide range of studies under submaximal conditions by using the steady-state rate of oxygen uptake at a given running velocity. C was found to be essentially constant at any speed between 10 and 20 km/h (e.g., Ref. 11). Van Ingen Schenau et al. (25) used a value around  $4.25 \text{ J} \cdot \text{kg}^{-1} \cdot \text{m}^{-1}$ , but, above resting, it should be  $4 \text{ J} \cdot \text{kg}^{-1} \cdot \text{m}^{-1}$  (11, 15). It is not yet known whether C remains the same at running velocities >20 km/h.

Caero ( $\text{J} \cdot \text{kg}^{-1} \cdot \text{m}^{-1}$ ) is  $k \cdot V^2$ , and, consequently, the power lost to air friction during sprinting is  $k \cdot V^3$  (in W/kg), where k is calculated from the values of air density ( $\rho$ ; in  $\text{kg}/\text{m}^3$ ), frontal area of the runner (Af; in  $\text{m}^2$ ), and Cd, according to  $k = 0.5 \cdot \rho \cdot \text{Ap} \cdot \text{Cd}$ . The  $\rho$  is calculated by using barometric pressure (Pb; in Torr) and air temperature ( $T^\circ$ , in  $^\circ\text{C}$ ), according to  $\rho = \rho_0 \cdot \text{Pb} \cdot 760^{-1} \cdot 273 \cdot (273 + T^\circ)^{-1}$ , where  $\rho_0 = 1.293 \text{ kg}/\text{m}^3$  is the  $\rho$  at 760 Torr (101.3 kPa) and  $273^\circ\text{K}$ .

Af was obtained by using the runner's mass (in kg) and height (in m) according to  $\text{Af} = (0.2025 \cdot \text{height}^{0.725} \cdot \text{mass}^{0.425}) \cdot 0.266$ .

The value of Cd (0.9) is the same as that of van Ingen Schenau et al. (25).

For our calculation of k, both the mass and height of the male and female runners (75 kg and 1.75 m and 64 kg and 1.78 m, respectively) were used. Also introduced in the calculation were the Pb (760 Torr) and  $T^\circ$  in Athens ( $25^\circ\text{C}$ ). The very limited effect of air humidity on  $\rho$  was not considered.

The men's 100-m final was run with a +0.2 m/s assisting wind (w), whereas in the women's final,  $w = +0.4$  m/s. Therefore, the power lost because of air friction was calculated as  $k \cdot (V - w)^3$ , and Caero was calculated as  $k \cdot (V - w)^2$ , where  $V - w$  is the runner's relative velocity to the air.

$\eta$  Of converting metabolic to external work. Van Ingen Schenau et al. (25) pointed out the necessity to consider  $\eta$  in power-balanced models. Di Prampero and colleagues (10, 11) and Peronnet and colleagues (21, 22) made the same choice, although Ward-Smith and colleagues did not (26, 27). In the present model, according to others (10, 11, 21, 22, 25), it was considered that Caero and Ckin should include a numerical value of  $\eta$ . Caero in demand is  $\text{Caero} = \eta^{-1} \cdot k \cdot V^2$  and  $\text{Ckin} = \eta^{-1} \cdot [0.5V_{(t+1)}^2 - 0.5V_t^2]$ . In the energetics models, which use the average velocity (11, 21, 22), it is usually considered that, during the acceleration phase of the sprint, the  $\eta$  for the transformation of metabolic energy into kinetic energy is roughly  $\eta = 0.25$ . Indeed, no recovery of elastic energy has

been considered during this early phase in which the running velocity is low, and consequently the overall running  $\eta$  must approach the  $\eta$  of muscular contraction (11). Conversely, in association with Caero, it is usually considered that, when speed increases,  $\eta$  can reach 0.5 partly because of the storage and recoil of elastic energy in exercising muscles (11).

With the above considerations in mind, the iterative procedure used in the present study offers the opportunity to include in the calculation of Caero and Ckin a value of  $\eta$  that increases in a linear way depending on instantaneous running velocity. The value of  $\eta$  might increase from 0.25 within the first movements of "pushing power" to 0.5 at maximal running velocity ( $V_{max}$ ; in m/s) because "reactive" power increases progressively. Thus  $\eta$  at any instant  $t$  is obtained by  $0.25 + (0.25 \cdot V_{max}^{-1}) \cdot V_t$ , where  $V_{max}$  was 11.8 m/s for the MWC and 10.7 m/s for the FWC.

*Models.* The detailed calculation made with the model we used, model  $V_t$ , is

$$\text{MAP}(1 - e^{-t/\tau_1}) + P_{max} \cdot e^{-t/\tau_2} = C \cdot V_t + \eta_t^{-1} \cdot k \cdot V_t^3 + \eta_t^{-1} (0.5V_{t+1}^2 - 0.5V_t^2) \cdot \Delta t^{-1} \quad (4)$$

where  $\eta$  varies over the range from 0.25 to 0.5 as a function of  $V_t$  as indicated above. Equation 4 is a nonlinear differential equation, which was solved numerically and by using an iterative procedure; the numerical integration was performed in steps of 0.01 s (25).

This mathematical model was compared with the model that uses model  $d/t$

$$\text{MAP} - (\text{MAP}\tau_1 \cdot t^{-1}) \cdot (1 - e^{-T/\tau_1}) + P_{max}\tau_2(1 - e^{-t/\tau_2}) \cdot t^{-1} = C \cdot V + \eta_a^{-1} \cdot k \cdot V^3 + \eta_b^{-1} \cdot 0.5 \cdot V^3 \cdot d^{-1} \quad (5)$$

where, in supply,  $\text{Paer}_t = \text{MAP}(1 - e^{-t/\tau_1})$  and  $\text{Pana}_t = P_{max} \cdot e^{-t/\tau_2}$  have been integrated with respect to time between 0 and the duration of the race ( $t$  in s), and where, in demand, C and k are defined above,  $\eta_a = 0.50$ ,  $\eta_b = 0.25$ ,  $d$  is the running distance ( $d = 100$  m), and  $V = d/t$ .

*Measurements.* The speed-time curves of the 100-m MWC and 100-m FWC were obtained from laser measurements (4). In brief, laser apparatus (LAVEG Sport, Jenoptik, Jena, Germany) were placed 15 m behind the start line at a height of  $\sim 1.7$  m. The laser beams were directed to the lower part of the runner's back in the upright position. Consequently, measurements were not possible during the first instants of the race when the runners were crouched in their starting blocks. Speed values were obtained from that instant when the athlete began to raise the trunk until he/she had crossed the finish line (Figs. 1 and 2). Any error due to the difference in height of the system in relation to the height of the lower back is considered insignificant.

The system operated at 50 Hz and measured the distance covered by the runner every 0.02 s. The system was scaled by using the exact metric distance between laser apparatus and start line (15 m) and finish line (115 m). Video cameras operating at 50 Hz were placed perpendicular to the running direction on the upper stands at the 30-, 50-, and 60-m line, which allowed the distance-time results at regular 10-m intervals over 100 m to be measured (Figs. 1 and 2). Laser and video measurements were compared, and there was a  $0.10 \pm 0.06$ -m ( $n = 10$ ) average difference between video and laser measurements for MWC and  $0.09 \pm 0.06$  m ( $n = 10$ ) for FWC.

Recorded speed curves of the MWC and FWC were fitted by using the differential Eq. 4 and the least-square method. In Eq. 4, five parameters had values fixed on the basis of prior knowledge as detailed in sections *Supply in the model*

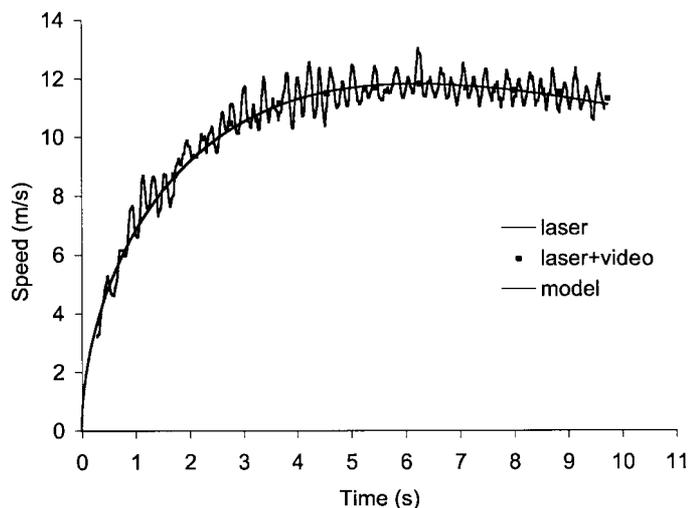


Fig. 1. Measured and predicted speed curves of male 100-m world champion (MWC). Squares are 10-m-interval video measurements.

and Demand in the model; they are MAP,  $\tau_1$ , C, k, and  $\eta$ . Two parameters were allowed to float free, Pmax and  $\tau_2$ , so that the solver in Microsoft Excel could calculate the optimal values.

## RESULTS

**Running performances.** The 100-m sprint finals at the Sixth IAAF World Championships in Athens 1997 were won by Maurice Greene (MWC), who recorded 9.86 s ( $w + 0.2$ ) including a 0.134-s reaction time, and Marion Jones (FWC), who recorded 10.83 s ( $w + 0.4$ ) including a 0.160-s reaction time.

**Speed curve modeling.** Figures 1 and 2 show the predicted and calculated speed curves obtained by laser and video measurements of the 100-m champions. The better fit with model  $V_t$  (Eq. 4) was obtained with Pmax = 90.7 W/kg and  $\tau_2 = 12.1$  s for MWC and Pmax = 76.2 W/kg and  $\tau_2 = 13.2$  s for FWC.

**Energy cost of running.** Average velocity for MWC and FWC were, respectively,  $100 \text{ m} \cdot (9.86 \text{ s} - 0.134 \text{ s})^{-1} = 10.28 \text{ m/s}$  and  $100 \text{ m} \cdot (10.83 \text{ s} - 0.160 \text{ s})^{-1} = 9.37 \text{ m/s}$ .

By using model  $d/t$ , the energy cost due to acceleration (Table 2) was obtained according to  $C_{kin} = \eta_b^{-1} \cdot 0.5 \cdot V^2 \cdot d^{-1}$ , where  $\eta_b = 0.25$ ,  $d = 100 \text{ m}$ , and  $V = 10.28 \text{ m/s}$  for MWC and  $V = 9.37 \text{ m/s}$  for FWC.

Aerodynamic cost (Table 2) was obtained according to  $Caero = \eta_a^{-1} \cdot k \cdot (V - w)^2$ , where  $\eta_a = 0.50$  and  $k = 0.0036 \text{ m}^{-1} \cdot \text{kg}^{-1}$  for MWC and  $k = 0.0040 \text{ m}^{-1} \cdot \text{kg}^{-1}$  for FWC;  $V$  was 10.28 and 9.37 m/s for men and women, respectively, and  $w = 0.2 \text{ m/s}$  for MWC and  $w = 0.4 \text{ m/s}$  for FWC. The contributions to demand of C, Caero, and  $C_{kin}$  are presented in Table 2.

By using model  $V_t$ , the instantaneous changes in both  $C_{kin}$  and Caero were obtained (Figs. 3 and 4). To compare Caero and  $C_{kin}$  with the values obtained with model  $d/t$ , those instantaneous values were averaged (Table 2).

$C_{kin}$  is the difference between the energy required to accelerate the body in the early phase of the race and the energy recovered during the final deceleration

phase. The former amounted to 171 J/kg in MWC and 140 J/kg in FWC, and the latter to 17 J/kg in MWC and 18 J/kg in FWC. This means that 10–12% of the energy of acceleration is recovered before the end of the race.

$C_{kin}$  was overestimated with model  $d/t$  by 37% in MWC and by 44% in FWC (Table 2). As a consequence, model  $d/t$  predicted 1) Pmax 20 and 14% higher than model  $V_t$  in MWC and FWC, respectively; 2) a higher decrease in the rate of anaerobic energy release; and 3) an 8% higher predicted contribution of anaerobic metabolism.

Caero was underestimated by 14% in both champions with model  $d/t$ .

## DISCUSSION

The present analysis of supply and demand during sprint running was based on the speed-time curves of MWC and FWC 100-m world champions recorded by laser apparatus on the occasion of the 1997 IAAF World Championships (4). A mathematical model, which uses the recorded instantaneous velocities, model  $V_t$ , was employed to describe the components of supply and demand. Results were compared with another model, which uses the average velocity (21).

**Model  $V_t$ : sensitive parameters.** The model we used includes seven parameters (Eq. 4). Five of them, MAP,  $\tau_1$ , C, k, and  $\eta$ , have numerical values fixed on the basis of prior knowledge, and only two, Pmax and  $\tau_2$ , were allowed to float free. To assess the importance of each parameter into the model, a sensitivity analysis was undertaken. By using supply characteristics obtained for MWC and FWC, the effects on predicted performance of a  $\pm 10\%$  change to the initial value of each parameter was tested.

First, even the largest  $\pm 10\%$  variation in MAP and  $\tau_1$  resulted in only minor changes in predicted performance (0.02 s in MWC and 0.03 s in FWC). This is consistent with the small contribution (5%) of aerobic metabolism to energy supply obtained, according to other studies (21, 22, 25–27).

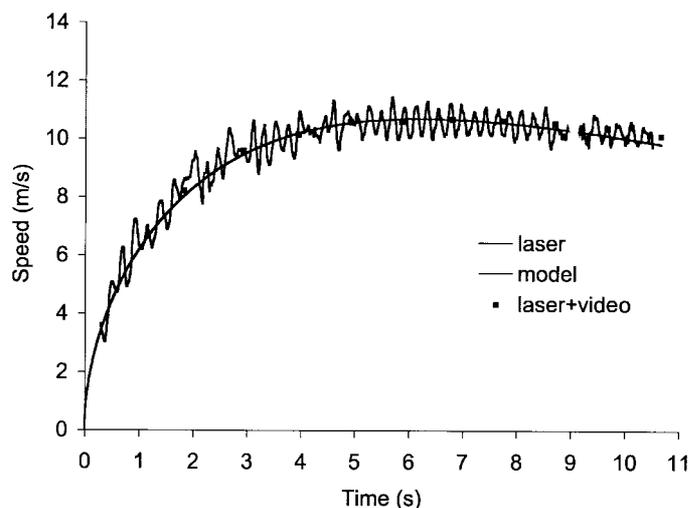


Fig. 2. Measured and predicted speed curves of female 100-m world champion (FWC). Squares are 10-m-interval video measurements.

Table 2. Supply and demand values in MWC and FWC

	Supply				Demand					
	aer, J/kg	ana, J/kg	Pmax, W/kg	$\tau_2$ , s	C, J/kg	Caero, J/kg	Ckin, J/kg	C*, J·kg <sup>-1</sup> ·m <sup>-1</sup>	Caero*, J·kg <sup>-1</sup> ·m <sup>-1</sup>	Ckin*, J·kg <sup>-1</sup> ·m <sup>-1</sup>
<i>MWC</i>										
Model $V_i$	30 (5%)	607 (95%)	90.7	12.1	400 (63%)	83 (13%)	154 (24%)	4.00	0.80	3.02
Model $d/t$	30 (4%)	654 (96%)	109.3	9.2	400 (58%)	73 (11%)	211 (31%)	4.00	0.73	2.11
<i>FWC</i>										
Model $V_i$	35 (6%)	557 (94%)	76.2	13.2	400 (67%)	71 (12%)	122 (21%)	4.00	0.70	2.45
Model $d/t$	35 (6%)	605 (94%)	87.0	11.5	400 (62%)	64 (10%)	176 (28%)	4.00	0.64	1.76

Percentages of total supply or total demand are shown in parentheses. MWC, male 100-m world champion; FWC, female 100-m world champion; aer, aerobic; ana, anaerobic. \*Mean of instantaneous values obtained with model  $V_i$ .

Figures 5 and 6 show how the final time could be affected by the other five parameters of Eq. 4. This analysis clearly demonstrates that Pmax has the greatest influence on the predicted running time, followed by C,  $\eta$ ,  $\tau_2$ , and to a lesser extent k. As a consequence, the supply-demand approach is really informative only if it can be ensured that Pmax, C,  $\eta$ , and  $\tau_2$  are reasonably assigned or predicted.

*Parameters with assigned values.* The assigned values for C and  $\eta$  have been plausibly derived from previous literature (e.g., Ref. 11). However, the values are partly unsecured. This is probably the weakest area of such mathematical models.

First, the numerical value for C was fixed on the basis of experimental data obtained at submaximal running speeds, although much higher speeds are concerned in our 100-m sprint model. Second, one should consider the great variability of C among different subjects in submaximal speeds. For example, in a group of middle-distance runners, Lacour et al. (16) found a range of variation of 20% according to previous

results (4, 24) over the range 3.5–4.2 J·kg<sup>-1</sup>·m<sup>-1</sup>. Assuming C amounts to  $\leq 3.92$  rather than 4 J·kg<sup>-1</sup>·m<sup>-1</sup> in our model, it would be enough for MWC to set a new world record (Figs. 5 and 6). The reason is that C is 63–67% of demand in 100-m sprint (Table 2).

Concerning  $\eta$ , there is no evidence in the literature that  $\eta$  might be around 0.25 during the most propulsive phase of acceleration. Nevertheless, a recent study (6) supported the 0.25 value, noting a lack of correlation between leg stiffness and acceleration during sprinting. The authors argued that forward power rather than reactive power is required during the acceleration phase. Conversely, as leg stiffness correlates to maximal running velocity, this could be a further argument for assuming an  $\eta$  of  $\sim 0.5$  at high speeds.

An  $\eta$  of 0.228 was used in model  $V_i$  used by van Ingen Schenau and colleagues (25). However, we observed that the predicted speed curves in this work were far from those that have been obtained with laser measurement. If the speed of MWC is considered, assuming  $\eta = 0.228$ , the predicted Pmax will be 135 W/kg and  $\tau_2$  at 9.6 s.

According to Ward-Smith and colleagues (26, 27),  $\eta$  is not usually assigned to Caero and Ckin. They argued

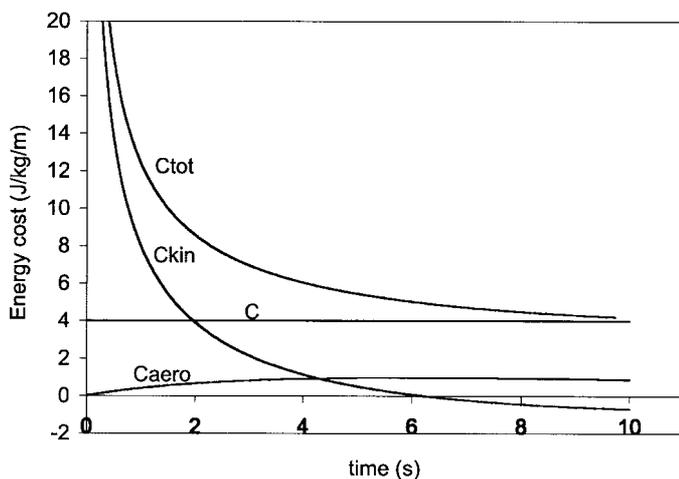


Fig. 3. Time course of the demand components in MWC. Total energy cost ( $C_{tot}$ ) =  $C_{kin}$  +  $C_{aero}$  + C, where  $C_{kin}$  is energy required to change kinetic energy,  $C_{aero}$  is the energy required to overcome air resistance, and C is the energy required to move forward.

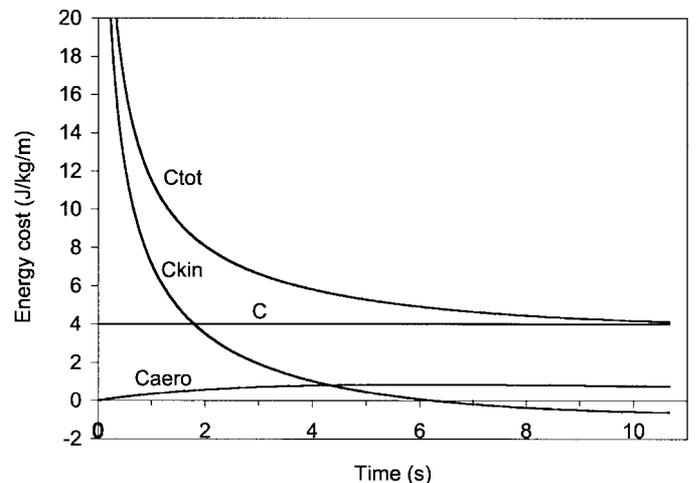


Fig. 4. Time course of the demand components in FWC.

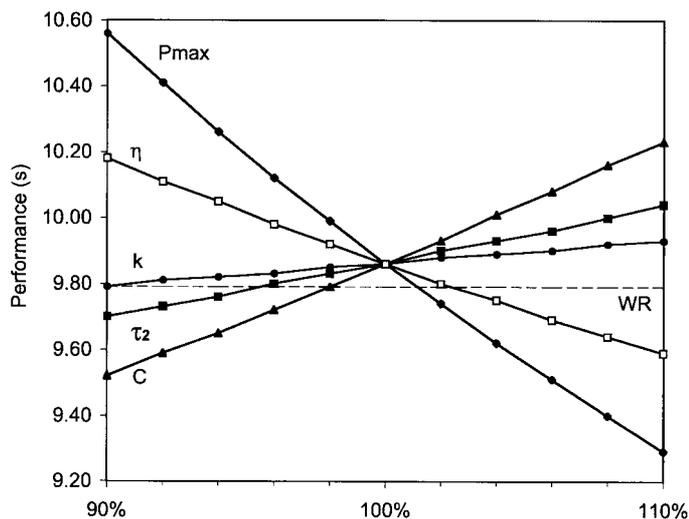


Fig. 5. Effect on the predicted performance of  $\pm 10\%$  changes in the initial values of five parameters in the velocity-time model ( $V_t$ ; Eq. 4) on MWC. Pmax, maximal anaerobic power;  $\eta$ , efficiency;  $k$ , air friction constant;  $\tau_2$ , parameter governing rate of anaerobic activity; WR, world record.

that the entire dissipation of thermal power is taken into account in the magnitude of  $C$ . Unfortunately, a standard value of  $C$ , e.g.,  $3.96 \text{ J}\cdot\text{kg}^{-1}\cdot\text{m}^{-1}$  (26) was used in this model so that no additional thermal energy concerning  $C_{\text{aero}}$  and  $C_{\text{kin}}$  can be taken into account. This leads to the predicted Pmax being only twofold MAP, which is not realistic in highly trained sprinters.

To conclude, there are many assumptions in the present model, particularly with regard to  $\eta$ .

**Predicted parameters: anaerobic metabolism.** Pmax and  $\tau_2$  were the two parameters allowed to float free in our model. This means that predicted values depend on other assumed parameters in the model. Pmax was predicted at around 90 and 75 W/kg in MWC and FWC, respectively (Table 2); at an  $\eta$  of 0.25, Pmax = 23 and 19 W/kg, respectively. This is very similar to the results of Pmax obtained in male sprinters by Van Ingen Schenau and colleagues (25) using cycling experimentation. Additionally, in sprinters of national level clocking around 10.6 s for men and 11.6 s for women, cycling Pmax measured in free-accelerated conditions amounted to 20 and 17 W/kg, respectively (1). These results are a first argument against using model  $d/t$  for sprint analysis because this model predicts a much higher Pmax (Table 2).

Again, Ward-Smith et al. (26) obtained values of Pmax that do not compare with those presently obtained because his mathematical framework did not include  $\eta$ . In the present study, Pmax (75–90 W/kg) was about four to five times higher than MAP (18 W/kg), which is realistic for highly trained sprint runners.

Although there is a consensus in the literature about using  $\tau_2$  for  $\sim 30$  s in models (21, 22, 25, 26), lower time constants for anaerobic energy release were presently predicted in world champions (Table 2). First, this rapid decrease in anaerobic energy supply is a consequence of  $\eta$  increasing rapidly to 0.5 so that the balanced external work in demand is moderate. Assuming

$\eta$  is correct, there are many arguments for present values of  $\tau_2$  around 12–13 s as realistic for elite sprint runners. For instance, it has been observed that the higher the initial maximal power, the higher the decline in power in the following seconds of exercise (18). This is mostly due to the great participation of the glycolysis in the first few seconds of maximal exercise (2, 12) and the inhibitory effect of proton on phosphofructokinase. As MWC and FWC both had high predicted Pmax, it is not surprising that the anaerobic rate decreased quickly.

By using a monoexponential model to describe anaerobic, energy release is approximate. Anaerobic energy supply relies actually on different components (stored phosphagens, stored oxygen, and glycolysis), each within their own, different time constants (27). The following may be reasonable estimates of time constants for each component: phosphagens = 9 s, stored oxygen = 3 s, and glycolysis = 35 s (e.g., Ref. 8). When most of the measured rates of release are for longer periods, typically 30–210 s (19, 25), the overall time constant will be dominated by the rate of release of glycolytic energy. Over shorter periods, the more rapid components will dominate. So, fitted over 10 s, time constants are likely to be shorter than when applied to longer periods of exercise. Additionally, by allowing for a faster rate of release in specifically trained athletes, and particularly a faster rate of release of energy from stored phosphagens, an overall time constant of 12–13 s seems very likely.

Taking into account the predicted Pmax and  $\tau_2$  (Table 2), we obtained an anaerobic contribution to energy supply amounting to 607 and 557 J/kg in MWC and FWC, respectively. On the basis of the direct metabolic approach of anaerobic ATP turnover in 6 and 10 s of maximal exercise (2, 12), these values might be underestimated. By making space for ATP derived from oxygen stores (i.e., attached to hemoglobin and myoglobin and available in the lungs), which probably amounts to 6 ml/kg (120 J/kg) (19) and has a rapid half

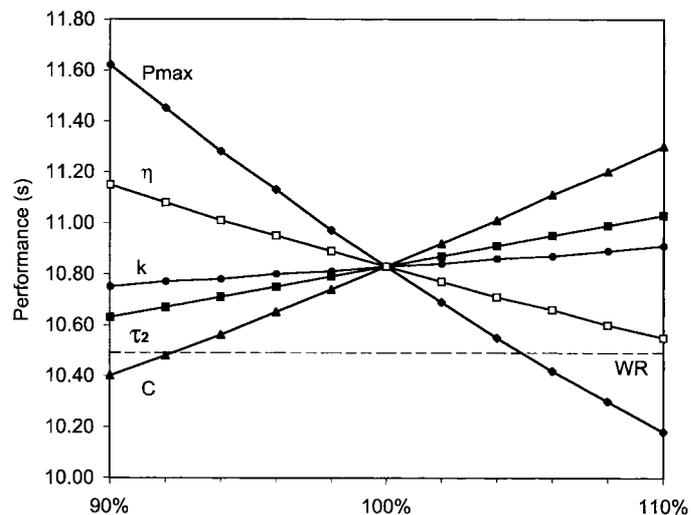


Fig. 6. Effect on the predicted performance of  $\pm 10\%$  changes in the initial values of five parameters in model  $V_t$  (Eq. 4) on FWC.

time in use, the strictly anaerobic ATP turnover might be 490 J/kg for MWC and 440 J/kg for FWC. However, direct examination of muscle metabolites (2) provided 130 mmol ATP turnover ( $\pm 5,200$  J) per kilogram active muscle mass (5,200 J/kg) in 10 s. By assuming that a quarter of the body mass is highly active during sprinting, this leads to an ATP turnover of 1,300 J/kg, a threefold higher value than that presently predicted in world-class 100-m runners.

Others have used postcompetition blood lactate concentrations as indicators of anaerobic energy expenditure during 400- and 800-m races (15). We are aware of post-100-m peak blood lactate concentrations ( $[\text{Lac}]_b$ ) noticeably different between Caucasian and African runners. In a group of Caucasian sprint runners with performance times of 10.54–10.69s,  $[\text{Lac}]_b$  reached 14.6–16 mM (17), whereas in Africans with an average time of 10.70s,  $[\text{Lac}]_b$  reached only 8.5 mM (13). On the basis of calculations made by others (15, 19), the amount of energy yielded by the glycolysis in Africans could be  $\pm 500$  J/kg. Supply, obtained as the sum of oxygen stores (120 J/kg), glycolysis in Africans ( $\pm 500$  J/kg), and depletion in phosphagens ( $\geq 400$  J/kg), is higher than the supply predicted in our model. Nevertheless, we can conclude that our understanding of whole body anaerobic capacity is too limited. Perhaps modeling the running performance is the most appropriate approach to study anaerobic energy supply in brief maximal exercises as suggested by others (27).

**Energetics of sprint running.** The energetics of sprint and middle-distance running might differ for several reasons. First, because of the comparative high speeds sustained in sprint running, aerodynamic resistance is a nonnegligible component of demand. Second, and more importantly, the cost of acceleration ( $C_{kin}$ ) is most likely to be high in world-class sprint runners. This is because 1) the speed increases from 0 to 9 m/s in  $<1.8$  s in men and from 0 to 8 m/s in  $<1.9$  s in women and 2) the acceleration phase represents  $\sim 60\%$  of the race (4). Accordingly, the contribution of  $C_{kin}$  amounted to 20–25% of the total demand in our work (Table 2, Figs. 3 and 4). This means that the model used to estimate  $C_{kin}$  has to be well defined, as a small error in  $C_{kin}$  can lead to quite large discrepancies in both predicted supply and demand. This confirms that using model  $d/t$ , where  $C_{kin}$  is not correctly defined, discourages the modeling of the energetics of short running distances. There is, as yet, no reason to reject model  $d/t$  for distances  $\geq 0.8$  km on the basis of  $C_{kin}$  being  $\leq 3\%$  of demand. Unfortunately, it might not be possible to create a model for 200- and 400-m races. As both are not maximal exercises, modeling the anaerobics by using a simple first-order system is obviously not correct. Furthermore,  $C_{kin}$  might be incorrectly approximate with model  $d/t$ , taking into account its relative contribution to the energy demand. Finally,  $Caero$  could be of importance at such high speeds, but the effect of wind could be difficult to assess as the runner changes direction during the race.

The model  $V_t$  we used makes it possible to obtain instantaneous changes during the race in both  $C_{kin}$

and  $Caero$  (Figs. 3 and 4). Interestingly, after  $\sim 6$  s of running, both MWC and FWC begin to decelerate. This means that they are paying back kinetic energy so that part of  $Caero$  requires no more supply. Demand is almost entirely set by C, so that, in practical terms, the runner just has to replace his or her limbs in the correct position before contact with the ground and bounce on the track. Consequently, it is not surprising that muscle stiffness, evaluated by using vertical rebounds, usually correlates with running at maximal velocity (6, 17).

The estimated  $Caero$  amounted to 80 J/kg (Table 2). This is fairly plausible on the basis of wind tunnel or other experiments (7, 14). Any changes in body size, Cd, or  $\rho$  would affect  $Caero$  through changes in k, as shown by  $k = 0.5 \cdot \rho \cdot A_f \cdot Cd$  (see METHODS). We show that a  $\pm 10\%$  change in k leads to  $\sim 0.07$  s advantage in running time (Figs. 5 and 6). Especially in world-class women, we noted some difference in height of comparable body mass. In the recent 2001 World Championship in Edmonton, Marion Jones (FWC in the present study; weight = 64 kg, height = 1.78 m) lost her title to Zhanna Pintussewich (weight = 64 kg, height = 1.64 m). On the basis of their 6% difference in  $A_f$ , the present model predicts a 0.05-s advantage to Pintussewich in still air or moderate assisting wind. However, with a head wind of 2 m/s, this advantage reaches 0.1 s. Height in sprint runners can be an advantage with regard to stride length, but on the basis of its role in  $Caero$ , it can be a disadvantage.

Differences in body mass affect the energetics of sprint running in a more complex manner. The supply-demand equation is balanced in J/kg body mass in our work. Therefore, any increase in body mass, which is less involved in power generation, would increase demand without changing supply. Increased upper body mass would increase both  $C_{kin}$  and  $Caero$  but in different proportions. We assessed that, in MWC, an additional 5 kg upper body mass would increase  $C_{kin}$  by  $\pm 5\%$  and  $Caero$  by 0.4%, resulting in a performance time of 10.03 s (Fig. 7), which is obviously the lower limit for a world-class running time. Inversely, suppos-

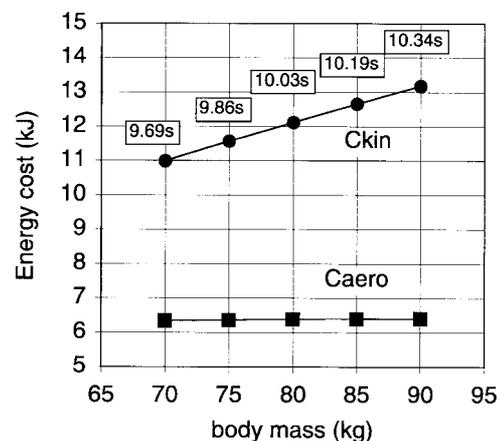


Fig. 7. Influence of body mass on  $C_{kin}$  and  $Caero$  (in kJ) and respective predicted performance times.

ing that upper body muscle mass has few other advantages, a 5-kg decrease resulted in a running time of 9.69 s (new world record) mainly due to the advantage of a 5% reduced  $C_{kin}$ .

More than body mass itself, the repartition of that mass seems to affect  $C$ . Previous investigations (20, 23) demonstrate that adding external masses to limb extremities widely increases the energy cost of moving forward. In sprinting events in which extremities are successively accelerated and decelerated at high rates, the lighter the extremities are, the lower  $C$  is.

To conclude the effect of body size on sprint-running performance, height presents no advantage in terms of energetics, but rather the runner's mass should power muscles that are as proximal as possible. It is interesting to note that runners increase in height and mass as their running distance increases from 60 to 400 m. It may be that  $C_{kin}$  is very important over short distances, as our analysis suggests, but as distance increases, size also confers other advantages, such as better reuse of elastic energy (6).

In conclusion, speed curves of world champions can be successfully used to model the energetics of sprint running. Our model provides realistic insights on components of both demand and supply, ensuring that  $P_{max}$ , the  $\eta$  of converting metabolic into external work, and  $C$  can be reasonably assumed or predicted. Unfortunately,  $\eta$  and  $C$  are unsecured, whereas  $P_{max}$  receives supporting results in the literature.

A MWC who runs 100 m in 9.86 s might yield 640 J/kg, including 95% anaerobic energy to balance an energy demand composed of 24%  $C_{kin}$  and 13%  $C_{aero}$ . The remaining cost of  $C$  in  $J \cdot kg^{-1} \cdot m^{-1}$  was supposed to be the same as that required in running middle or long distances. A high initial level of power output, amounting to at least 90 W/kg in men and 75 W/kg in women, might be a prerequisite for top-level results. Little is known about the rate of decrease in maximal power during the race. A time constant for anaerobic energy release, amounting to 12–13 s, has not been confirmed. Neither height nor body mass seems to have any advantage in the energetics of sprint running.

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